

The laminar–turbulent transition in a fibre laser

E. G. Turitsyna¹, S. V. Smirnov², S. Sugavanam¹, N. Tarasov¹, X. Shu¹, S. A. Babin^{2,3}, E. V. Podivilov^{2,3}, D. V. Churkin^{1,2,3}, G. Falkovich^{4,5} and S. K. Turitsyn^{1,2*}

Studying the transition from a linearly stable coherent laminar state to a highly disordered state of turbulence is conceptually and technically challenging, and of great interest because all pipe and channel flows are of that type^{1,2}. In optics, understanding how a system loses coherence, as spatial size or the strength of excitation increases, is a fundamental problem of practical importance^{3–5}. Here, we report our studies of a fibre laser that operates in both laminar and turbulent regimes. We show that the laminar phase is analogous to a one-dimensional coherent condensate and the onset of turbulence is due to the loss of spatial coherence. Our investigations suggest that the laminar–turbulent transition in the laser is due to condensate destruction by clustering dark and grey solitons. This finding could prove valuable for the design of coherent optical devices as well as systems operating far from thermodynamic equilibrium.

Nature does not allow us to increase the size of a system without eventually losing coherence. For example, although a coherent laminar flow through a pipe is always linearly stable, increasing the pipe diameter or speed eventually makes the flow turbulent, which vastly increases drag^{1,2,6–9}. One needs to identify the origins of the onset of turbulence to understand and control it. This is a formidable task for a linearly stable system due to the lack of a linear instability analysis, which shows what destroys the laminar state and helps identify the patterns that appear instead^{2,6–9}.

In optical fibres with normal dispersion, a coherent monochromatic wave or spectrally narrow packets are linearly stable with respect to modulation instability¹⁰. In a laser cavity with normal dispersion, it is theoretically possible to overcome wave dephasing by nonlinear four-wave mixing and achieve a classical wave condensation to form a coherent state^{11,12}. A kinetic condensation of classical waves was recently observed in two-dimensional, optical Hamiltonian systems, revealing a link with Bose–Einstein condensation, including condensation of photons^{13,14}. However, operational regimes in many fibre lasers are characterized by very irregular light dynamics and a low degree of coherence. A quasi-continuous-wave (quasi-c.w.) fibre laser normally generates so many modes (up to 1×10^6) that fluctuations in their amplitudes and phases result in stochastic radiation, which calls for description in terms of wave turbulence^{15–19}. To establish conditions for the existence of a coherent condensate and reveal the mechanisms of coherence loss, it is critically important to comprehensively study the laminar–turbulent transition in fibre laser radiation, as is done in classical hydrodynamics experiments².

In our experiments, increasing the cavity length or the power of a fibre laser causes the output to pass from a coherent laminar state to a turbulent one. Having a laminar–turbulent transition in an optical system makes it possible to investigate fundamental questions of non-equilibrium operation in lasers such as ‘What are the

mechanisms of losing coherence in fibre lasers?’ and ‘Is the transition due to an increase in temporal or spatial complexity?’.

The Methods briefly describes the experimental set-up. Here, we define ‘space’ and ‘time’ in our system to allow for meaningful comparison between fibre optics and hydrodynamics. The radiation intensity I is measured in a single point as a function of time. As light makes round trips in the resonator, the radiation is measured within a series of time windows separated by the round-trip time τ_{rt} . The result is a function of a continuous variable within a window, denoted t , and a discrete variable, $T = N \times \tau_{rt}$, where N is the number of round trips. The fastest process is linear propagation with the speed of light c , so the t dependence by the transform $t - x/c$ represents the dependence on the spatial coordinate x along the resonator. The spectra of the radiation are obtained by performing a Fourier transform over t . Energy pumping, dissipation, dispersion and nonlinearity lead to a slow evolution of the spectra over many round trips T . In other words, the slow evolution coordinate T has the meaning of *time*, while the fast time t is equivalent to the longitudinal spatial coordinate x .

To observe the laminar–turbulent transition, the laser power was changed. The laminar regime is realized at low power and the turbulent regime at a high pump power. There is a sharp transition in the properties of the laser radiation on increasing the power. The optical spectrum width Γ increases almost twofold with an increase in power of only 1% (Fig. 1a). At the same power, a sharp transition also occurs in the most probable intensity (Fig. 1b). Below the transition, generation is quite stable and the intensity fluctuations are small. The intensity probability density function (PDF) has a sharp narrow peak (Fig. 1b, inset) centred at the mean intensity, as it should for a coherent state. Just before the transition, the peak widens slightly, but the mean intensity remains most probable. At the transition, the most probable intensity falls by almost a factor of two, while the PDF changes form and develops a wide, approximately exponential tail that is a manifestation of a significant probability of high-intensity fluctuations. The transition is also detected as a drop in the background level of the intensity autocorrelation function (ACF) from a coherent-state level to a stochastic regime (Fig. 1c). Intensity time traces just before and after the transition are shown in Supplementary Fig. S3.

The transition corresponds to the loss of coherence in the system. The total number of generated modes M is given by $M = \Gamma \times (2Ln/c) \approx 1 \times 10^5$, so the laminar state is fundamentally different from single frequency (single longitudinal mode) generation^{20,21}. Here, n is the refractive index and L is the fibre length. The spectrum is only two times wider after the transition, yet the spatiotemporal dynamics of radiation are very different in the laminar and turbulent regimes. Figure 2a,b shows rather small fluctuations before the transition, and recurring spatiotemporal patterns after the transition. We detected long-living propagating intensity minima both

¹Aston Institute of Photonic Technologies, Aston University, Birmingham B4 7ET, UK, ²Novosibirsk State University, Novosibirsk 630090, Russia, ³Institute of Automation and Electrometry, Siberian Branch, Russian Academy of Sciences, Novosibirsk 630090, Russia, ⁴Weizmann Institute of Science, Rehovot 76100, Israel, ⁵Institute for Information Transmission Problems, Moscow 127994, Russia. *e-mail: s.k.turitsyn@aston.ac.uk

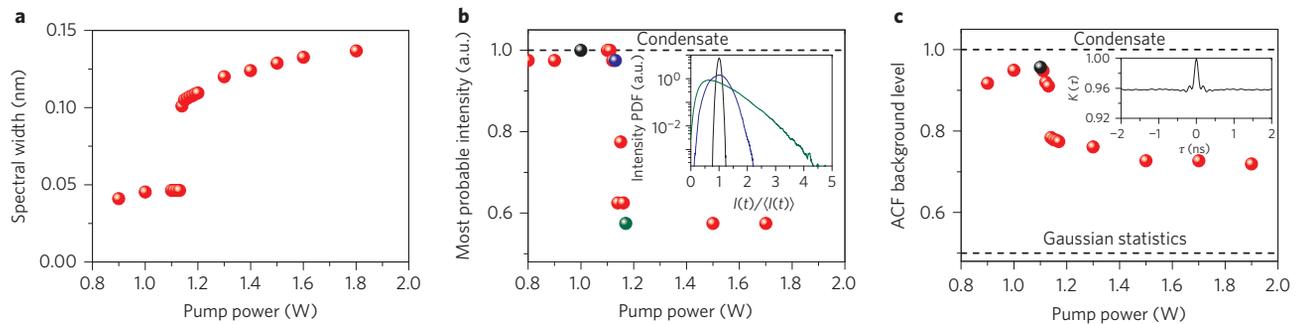


Figure 1 | Laminar-turbulent transition in the fibre laser experiment. **a**, Optical spectrum width (proportional to the number of excited modes) versus power. **b**, Most probable intensity versus power, and the full-intensity PDFs before and after the transition (inset). Colours link curves in the inset to points on the main graph. **c**, Background level of the intensity autocorrelation function (ACF), $K(\tau) = \langle I(t, T) \times I(t + \tau, T) \rangle$, measured at large τ . Inset: typical ACF before the transition. For a coherent state, $K(\tau) \rightarrow 1$. For completely stochastic radiation with Gaussian statistics, $K(\tau) \rightarrow 0.5$.

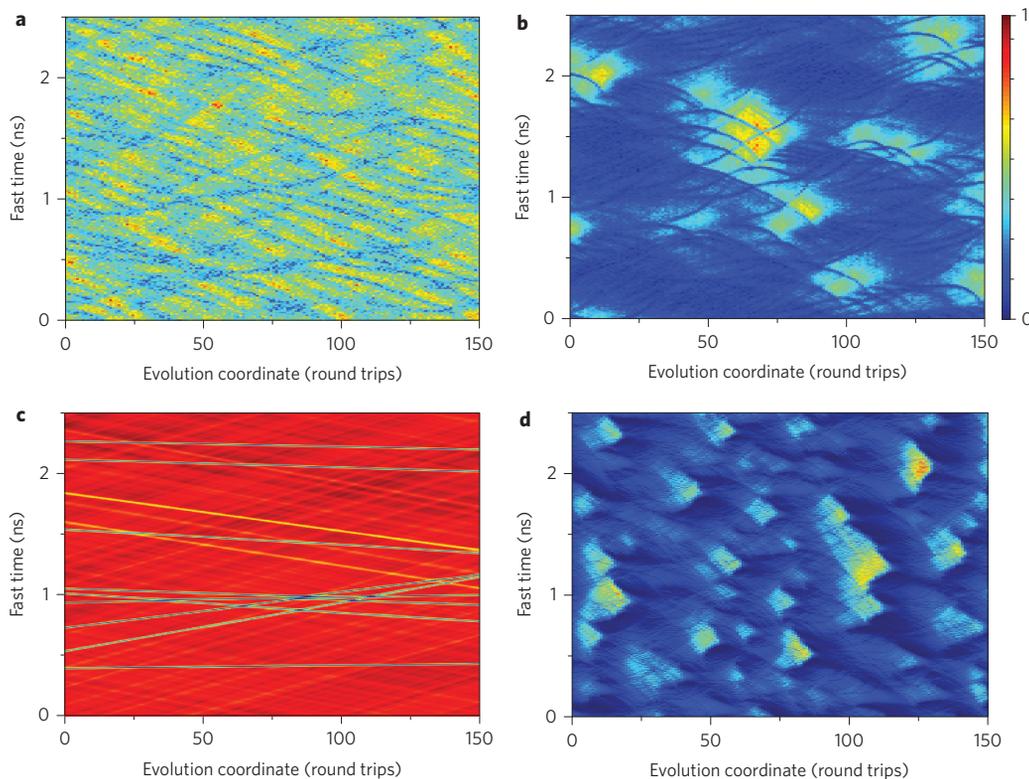


Figure 2 | Coherent structures in spatiotemporal dynamics in experiment and numerical simulation for laminar and turbulent regimes. **a-d**, Space-time diagram of intensity $I(t, T)$ for the laminar regime in the experiment (**a**), the turbulent regime in the experiment (**b**), the laminar regime in modelling (**c**) and the turbulent regime in modelling (**d**). Evolution coordinate $T = N \times \tau_{rt}$ is shown in terms of round-trip number N .

on a stable laminar background (Fig. 2a) and on a strongly fluctuating turbulent background (Fig. 2b). As the typical nonlinear length $L_{NL} = 1/(\gamma l) \approx 1$ km for the transition power (γ is a nonlinear coefficient), these structures live for ~ 100 nonlinear lengths, so they are coherent. The temporal width of the coherent structures is at the limit of our experimental resolution.

To resolve the internal details of the coherent structures observed, we used modelling based on the generalized, scalar, nonlinear Schrödinger equation (NSE). (For more details see Supplementary Section 'Numerical modelling'¹⁰.) Although the NSE is comparable to the Navier-Stokes equation, describing the fluid flow in terms of universality, deceptive simplicity and sheer beauty, the former is much more amenable to numerical treatment. The NSE is commonly used to describe coherent structures and stochastic-driven processes in optical fibres and fibre lasers^{11,22-27}.

For our fibre laser, modelling demonstrates the same laminar-turbulent transition at comparable levels of pump power. Moreover, the numerical simulations demonstrate that the laminar state is a coherent condensate and the transition is condensate destruction. Indeed, a coherent condensate must support long acoustic waves satisfying the Bogoliubov dispersion relation $\omega \propto k$, in distinction from usual dispersive waves where $\omega \propto k^2$ without the condensate²⁸. We found that the spatiotemporal spectrum $I(k, \omega)$ had maxima along straight lines in the laminar regime and along parabolic lines after the transition (Supplementary Fig. S5). Numerical simulations reveal spatiotemporal coherent structures similar to those observed experimentally (Fig. 2c,d). Remarkably, their shape and phase shifts are well described by the analytical form of dark and grey solitons (Supplementary Fig. S6), which are analytical solutions of the one-dimensional NSE²⁹.

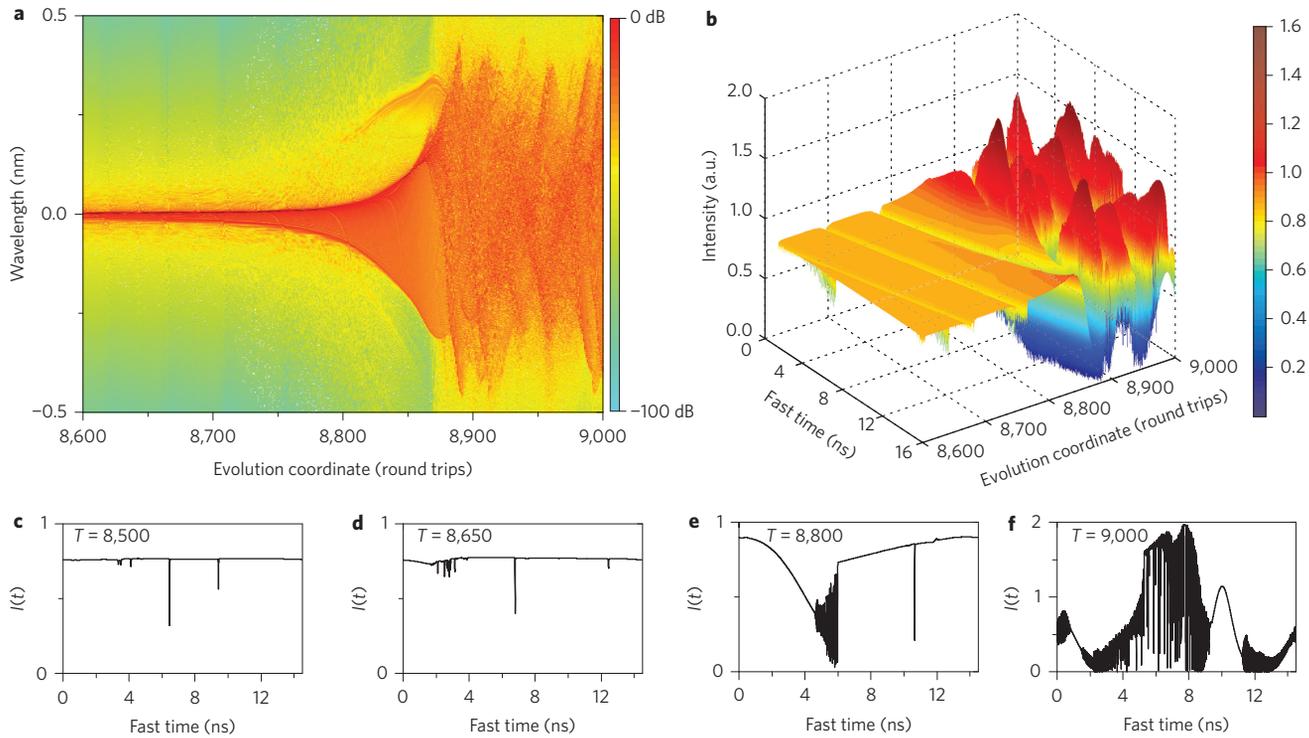


Figure 3 | Soliton clustering at the laminar-turbulent transition (numerical modelling) at fixed power. **a**, Radiation spectrum $I(\lambda)$ in a logarithmic scale versus evolution coordinate T . **b**, Radiation intensity $I(t)$ versus T . A bunch of solitons creates a deepening minimum moving with a negative speed along t (made into a circle; in other words, the points $t = 0$ ns and $t = 15.7$ ns are the same). At $T \approx 8,800$ round trips, this minimum is deep enough to break the condensate into two pieces, after which the total breaks down into many pieces. **c–f**, Radiation intensity $I(t)$ at four different T : condensate with rare isolated solitons (**c**); beginning of soliton clustering (**d**); condensate breakdown (**e**); turbulence (**f**). The whole spectral and spatial evolution is provided in Supplementary Movie S1. Evolution coordinate $T = N \times \tau_{rt}$ is shown in terms of round-trip number N .

Modelling reveals the underlying mechanism of the laminar-turbulent transition in great detail and shows that the solitons are key to the onset of turbulence. With an increase in pump power or cavity length, more and more solitons are generated, which leads to turbulence. Figure 3 shows how the transition develops over the evolution coordinate T at a fixed power. The moment of transition is clearly seen by spectral widening in Fig. 3a and the breakdown of spatial coherence in Fig. 3b. Solitons proliferate and cluster, creating a deep minimum that breaks the condensate (Fig. 3c–e). We concluded that the laminar-turbulent transition, observed experimentally and modelled numerically, originates from the appearance, proliferation and clustering of solitons. In a linearly unstable system, solitons may appear as an outcome of instability as, for instance, in capillary-wave turbulence⁷.

Condensate destruction leads to the creation of an intermittent state with a rather narrow spectrum, but limited spatial coherence (along t). In contrast to the traditional (dynamic system) view that turbulence arises from an increase in temporal complexity, the spatial breakdown of coherence is the leading process here, similar to a pipe flow^{2,9}. Even when the asymptotic turbulent stage has not yet fully developed, the condensate has already broken into pieces. When the soliton density becomes high, and the condensate is filled with dips and voids, the state is barely distinguishable from a dense mixture of coherent bright structures (such as bright solitons and breathers). Spatiotemporal patterns expand and shrink (having a rhombic form in the t - T plane) with approximately the same velocity (the slope in x - t space) as the propagation of solitons on them (Fig. 2b,d). The patterns recur quasi-periodically and move as a whole against the background, which confirms their coherent nature (Supplementary Fig. S8). The intensity correlation function over the evolution

coordinate shows the statistical signature of the quasi-periodical recurrence.

Repeating the simulations, with the only difference being a small change in the initial noise, we found that the lifetimes of the condensate fluctuate strongly; that is, the laminar-turbulent transition

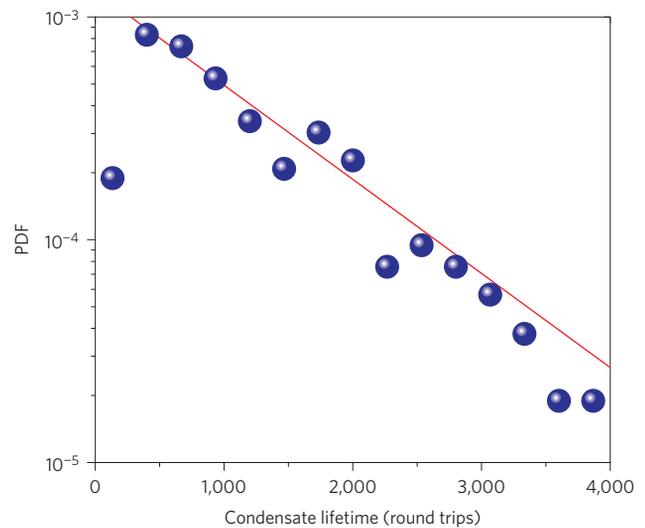


Figure 4 | Probability density function for the condensate lifetime showing the probabilistic nature of the laminar-turbulent transition via soliton clustering. The straight line is an exponential approximation at large lifetimes.

via soliton clustering is stochastic (c.f. ref. 2). The probability of surviving falls exponentially as in radioactive decay (Fig. 4). This suggests that, after some time, the probability of decay is constant in time and is independent of excitation time.

Flows are controlled by the Reynolds number, the ratio between nonlinear and linear terms in the Navier–Stokes equation. A similar ratio between nonlinearity and dispersion can be introduced and measured for a fibre laser, further developing the fluid–laser analogy (Supplementary Section ‘Definitions’).

In summary, we have observed the laminar–turbulent transition in fibre laser radiation and have identified the mechanism of this transition, opening new possibilities for studying the fundamental problem of turbulence onset in optical devices. We have discovered the critical role of coherent structures—dark and grey solitons—in destroying laser coherence, finding a link between solitons and turbulence; namely, localized coherent structures break long-range coherence. A useful analogy is thus found between laser and fluid flows. Both systems can lose spatial coherence via a transition that is of a probabilistic nature. We anticipate that our results will lead to better understanding of coherence breakup in lasers and the development of new optical engineering concepts and novel classes of lasers that operate in far-from-equilibrium regimes.

Methods

The fibre laser used in the experiments has a standard, all-fibre design with a cavity made specifically from a high-normal-dispersion fibre ($D = -44 \text{ ps nm}^{-1} \text{ km}^{-1}$, nonlinear coefficient $\gamma = 3 \text{ km}^{-1} \text{ W}^{-1}$) with a length of 770 m, placed between specially designed, all-fibre laser mirrors (fibre Bragg gratings). The mirrors have super-Gaussian spectral profiles of sixth order, $\sim 2 \text{ nm}$ bandwidth, with a dispersion variation of less than 10 ps per bandwidth. This is crucial for the experimental realization of the coherent laminar state and the transition to the turbulent state (see Supplementary Information for details). Fibre mirrors were written directly in a fibre core using an on-site fibre Bragg grating writing facility, following the refractive index longitudinal profile calculated numerically to obtain the desired spectral and dispersion response.

The spatiotemporal properties of the laser radiation were analysed using an oscilloscope with 36 GHz real-time bandwidth, comparable with the optical bandwidth of the radiation. Numerical modelling was based on two complementary approaches: (1) analysis of longitudinal, resonator-mode evolution with round trips and (2) computation of field dynamics using generalized NSEs (see Supplementary Information for details).

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Author contributions

S.K.T. and G.F. initiated the study. D.V.C. conceived the experiment and, with S.S. and N.T., carried it out. E.G.T. and S.V.S. designed and conducted the numerical modelling. E.G.T. and X.S. designed the special laser mirrors. X.S. fabricated the laser mirrors. G.F., S.K.T. and D.V.C. guided the theoretical and experimental investigations. G.F., S.K.T., D.V.C., E.V.P., S.A.B., S.V.S., E.G.T., S.S. and N.T. analysed the data. G.F., S.K.T. and D.V.C. wrote the paper.

Additional information

Supplementary information is available in the online version of the paper. Reprints and permissions information is available online at www.nature.com/reprints. Correspondence and requests for materials should be addressed to S.K.T.

Competing financial interests

The authors declare no competing financial interests.