On Saturable Absorbtion in Ultra-Long Mode-Locked Fiber Lasers

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Abstract: We propose a method to estimate analytically the saturable absorber output energy as a series of powers of the recovery time in order to improve the estimation of the output energy in ultra-long fiber lasers.

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1. Theoretical analysis

As it is known, the saturable absorbers (SA) are used to obtain the stable pulse generation in mode-locked lasers. In practice, it is useful to estimate how large the SA losses are for a given SA input signal. To estimate how the input signal evolves in a saturable absorber, we consider the following general equation [1]:

$$\frac{dq(t)}{dt} = -\frac{q(t)-q_0}{\tau} - \frac{qP(t)}{\tau P_{\text{sat}}}. \tag{1}$$

Here $P(t)$ is the SA input power, τ is the recovery time, P_{sat} is the saturation power, q_0 is the modulation depth, and $q(t)$ is the SA losses.

It has been derived that if $P(t)$ is an even function of time, the following takes place:

$$E_{\text{out}} = E_{\text{in}} - \int\limits_{\mathbb{R}} q(t)P(t)dt = E_{\text{in}} - q_0 \sum_{k=0}^{\infty} \tau^{2k} \int\limits_{\mathbb{R}} f_k(t)dt, \tag{2}$$

where $f_0(t) = P(t)\xi(t)$, $f_1(t) = [f_0(t)]'_t \xi(t)$, $f_2(t) = [f_1(t)]'_t \xi(t)$, ..., $f_n(t) = \frac{df_{n-1}(t)}{dt}\xi(t)$, and $\xi(t) = 1/(1+P/P_{\text{sat}})$. Particularly, equation 2 can be applied to any input function of practical importance, including the Gaussian input pulses and the hyperbolic secant input pulses.

To a first approximation, the output energy can be found as follows:

$$E_{\text{out}} \approx E_{\text{in}} - q_0 \int_{\mathbb{R}} \frac{P(t)}{1+P(t)/P_{\text{sat}}} dt. \tag{3}$$

The first approximation of the output energy does not depend on the SA recovery time. It can be shown that series 2 converges very fast and for many cases in practice it gives the good estimation of the SA output energy. Namely, it has been proven that the absolute error between the first approximation and series 2 does not exceed the following value:

$$\left| E_{\text{out}} - (E_{\text{in}} - q_0 \int_{\mathbb{R}} f_0(t)dt) \right| \leq \frac{2\tau P_0 q_0}{P_0/P_{\text{sat}}+1}. \tag{4}$$

2. Output energy estimation in ultra-long fiber lasers

Let us consider the dissipative fiber laser with the all-normal ring cavity. The scheme of the laser is presented in figure 1a [2]. The fiber laser consists of a passive fiber of variable length (of 500-2000 m in length), the Er-doped fiber of 2 m in length, the saturable absorber, and the output coupler. The parameters of a fiber laser are listed in [2].

The analytical method to estimate the output energy in such laser systems was initially proposed in [3] on the assumption that SA losses L_{SA} are varied between 0 and q_0, i.e. $0 < L_{\text{SA}} < q_0$. For the fiber laser under consideration, the output energy can be expressed as follows [3]:

$$E_{\text{out}} = E_{\text{in}} \exp(\alpha_P L_P) \cdot (1-R)/R. \tag{5}$$

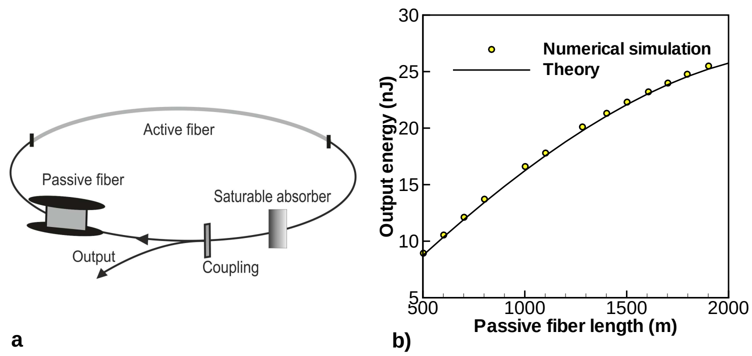

Fig. 1. a) The ring laser cavity scheme; b) The dependence of the output energy on the passive fiber length. The curve corresponds to the analytical results, and dots show the numerical results.

Here $\alpha_P L_P$ is the total loss in the passive fiber, R is the output coupler parameter, $s = \alpha_A/g_A$, and $S = (\alpha_A L_A + \alpha_P L_P + L_{\mathrm{SA,dB}} + R_{\mathrm{dB}})/(g_A L_A)$. The active fiber input energy E_{in} can be expressed as follows [3]:

$$E_{\mathrm{in}} = E_{\mathrm{sat}} \frac{1-s}{s} \exp(0.5G(s-S)) \cdot \frac{\sinh(0.5G(1-S)s)}{\sinh(0.5G(1-s)S)}. \tag{6}$$

The major drawback of formulae 5 and 6 is that they give the accurate results only if the SA losses are known. The theoretical results described above enable to improve the accuracy of formula 5, because analytical formula 3 gives the good approximation of SA losses.

However, to calculate SA losses, it is necessary to know the peak power and the waveform of an input pulse. For the long-cavity normal laser depicted in figure 1a, the dissipative soliton in the cavity preserves the waveform of the hyperbolic secant [4]. Particularly, we have the secant input at the saturable absorber. For a long-cavity laser $T(z)^2/\beta_2 > 1/(\gamma P(z))$, when the signal propagates along the passive fiber. Here β_2 is the second-order dispersion, and γ is the fiber nonlinearity coefficient. At the passive fiber input, however, the following is the case: $T^2/\beta_2 \approx 1/(\gamma P)$. Consequently, $E^2 \approx 4\beta_2 P/\gamma$. This relation makes possible to obtain the peak power P_0 as a function of E_{in}.

Using formula 3, it can be established analytically that if the input power $P(t) = P_0/\cosh^2(t/T_0)$, and $\xi = P_{\mathrm{sat}}/P_0$,

$$E_{\mathrm{out}} = 2P_0 T_0 - \frac{q_0 T_0 \cdot P_{\mathrm{sat}}}{2\sqrt{1+\xi}} \cdot \ln \frac{(\xi+2)(\sqrt{1+\xi}+2)-2}{(\xi+2)(\sqrt{1+\xi}-2)+2}. \tag{7}$$

To verify the theoretical results obtained above we have performed the complete numerical simulation of the laser system depicted in figure 1a. The details of the numerical simulation are provided in [2]. Figure 1b demonstrates the comparison of the output energy obtained through the full mathematical modeling based on the Nonlinear Schödinger equation with the analytical results obtained via formula 5. Using theoretical formula 7 for the hyperbolic secant input pulse, we are able to find the SA losses to a high degree of accuracy. The precise knowledge of SA losses allows the output energy to be estimated more accurately using formula 5. As it can be seen from figure 1b, the results are nearly identical for different lengths of the passive fiber. It should be noted that other laser systems (i.e. the dispersion-managed lasers) can be investigated in a similar way. The work was supported by the Russian Science Foundation (project 14-21-00110).

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