# Analytical Solutions for Power Evolution in the Effective Two-level Medium Laser Models

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**Abstract:** We present analytical solution describing power evolution in the general two-level active medium and the generated output power in the Fabry-Perot and ring fiber laser configurations.

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## 1. Introduction

Design of modern fiber lasers require massive numerical modelling because of multiple system parameters and complex nonlinear nature of light dynamics in the cavity. Therefore, analytical results for the light evolution in laser cavities are useful for optimisation and understanding of the underlying dynamics [1, 2, 4, 5]. We present here analytical solution for power evolution in general effective two-level active medium. This solution, can be used in a combination with the amplitude field modelling allowing to reduce by orders of magnitude the simulation time by eliminating the first iterative procedure. We consider signal evolution in an effective two-level gain model (see e.g. [4]).

$$\frac{dP^{\pm}(z)}{dz} = \pm\alpha_p\left(\frac{P(z)+\frac{1}{\mu}S(z)}{1+P(z)+S(z)}-1\right)P^{\pm}(z); \quad \frac{dS^{\pm}(z)}{dz} = \pm\alpha_{\mathrm{s}}\left(\frac{\mu P(z)+S(z)}{1+P(z)+S(z)}-1\right)S^{\pm}(z), \tag{1}$$

where $\mu = \frac{\lambda_p}{\lambda_s}\frac{\alpha_p}{\alpha_s}\frac{P_p^{\mathrm{sat}}}{P_s^{\mathrm{sat}}}$, $\lambda_p$ and $\lambda_s$ are the central wavelengths of the pump and the signal, respectively, $\alpha_p$ and $\alpha_s$ are the unsaturated losses, plus and minus stands for forward and backward pump (index p) and signal (index s) waves, $P_p^{\mathrm{sat}}$ and $P_s^{\mathrm{sat}}$ are the saturation powers, $P^{\pm}(z) = P_p^{\pm}(z)/P_p^{\mathrm{sat}}$, $P(z) = P^+(z)+P^-(z)$, $S^{\pm}(z) = P_s^{\pm}(z)/P_s^{\mathrm{sat}}$, $S(z) = S^+(z)+S^-(z)$ with the boundary conditions $P^+(0) = P_0^+$, $P^-(L) = P_L^-$, $S^+(0) = S_0^+$, and $S^-(L) = S_L^-$. The gain is defined $G = S^+(L)/S_0^+$. Denote $\zeta = \frac{1}{\mu}\frac{\alpha_p}{\alpha_s}$ and $\psi = \frac{\mu-1}{\mu}\alpha_p$. It is easy to check that the following gain equation can be derived from the system 1:

$$\ln[G] + \left(S_0^+ + S_L^-\right)[G-1] + \frac{1}{\zeta}\left(P_0^+ + P_L^-\right)\left[G^{\zeta}\exp(-\psi L)-1\right] + \alpha_s L = 0. \tag{2}$$

Next we apply this general expression to determine the output power in two classical laser configurations: ring laser cavity (Fig. 1a) and Fabry-Perot laser configuration (Fig. 1b).

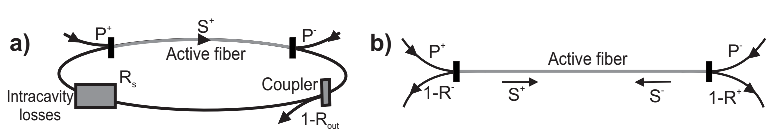


Fig. 1. a) The ring laser cavity; b) The Fabry-Perot laser cavity.

## 2. Ring cavity

Consider the ring laser with two external pumps: forward $P^+$ and backward $P^-$, the resonator includes the output coupler with the power transmission coefficient $R_{\mathrm{out}}$ (the out-coupled signal power is given by $1-R_{\mathrm{out}}$), and the active fiber of length $L$. Let us denote the total cavity loss (including out-coupled power) as $\Sigma$. The lasing condition requires that the total cavity gain $G$ compensates for all cavity losses $\Sigma$, ($G\Sigma = 1$, hence $G = 1/\Sigma$). For the output signal power at the lasing threshold we get: $P_{\rm out} = P_s^{\rm sat}(1-R_{\rm out})\cdot S^+(L) = P_s^{\rm sat}(1-R_{\rm out})S_0^+/\Sigma$. Combining this with Eq. 2 it is easy to find the output power at the threshold of generation:

$$P_{\rm out} = P_s^{\rm sat}\frac{1-R_{\rm out}}{1-\Sigma}\left\{\ln[\Sigma] + \frac{1}{\zeta}\left(P^+ + P^-\right)\left[1-\Sigma^{-\zeta}\exp(-\psi L)\right] - \alpha_s L\right\}. \quad (3)$$

The output power as a function of the total cavity losses $\Sigma$ and the out-coupling coefficient $1-R_{\rm out}$ is shown in Fig. 2a.

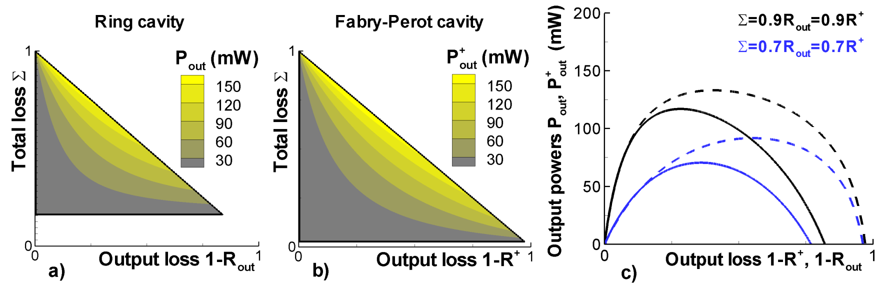


Fig. 2. The figures a) and b) show the dependence of the output power on the total loss and the out-coupling loss. The solid lines in figure c) correspond to $P_{\rm out}$ (the ring cavity), dashed lines correspond to $P_{\rm out}^+$ (Fabry-Perot cavity). The black and blue lines show the cases $\Sigma = 0.9R^+$ and $\Sigma = 0.7R^+$ respectively, where $\Sigma$ includes both the output losses and other cavity losses. The parameters of the Yb-doped fiber are the same as in [4], but $P^+ = 5$ W and the fiber length $L = 1$ m.

## 3. Fabry-Perot cavity

Next we consider the case of the Fabry-Perot cavity. Let us denote by $P^+$ and $P^-$ the external forward and backward pumps, and $R^+$ and $R^-$ the forward and backward signal reflections, respectively. $L$ is the active fiber length. In the Fabry-Perot cavity the total losses are $\Sigma = R^+\cdot R^-$, and the lasing condition gives $G = 1/\sqrt{\Sigma}$. For the output signal powers it is straightforward to derive that: $P_{\rm out}^+ = P_s^{\rm sat}(1-R^+)\cdot S^+(L) = P_s^{\rm sat}(1-R^+)S_0^+/\sqrt{\Sigma}$, $P_{\rm out}^- = P_s^{\rm sat}(1-R^-)\cdot S^-(0) = P_s^{\rm sat}(1-R^-)S_L^-/\sqrt{\Sigma}$. After simple algebra we get from Eq. 2 the following expression for two outputs powers (at the left and right edges):

$$P_{\rm out}^{\pm} = P_s^{\rm sat}\frac{1-R^\pm}{1-\sqrt{\Sigma}}\cdot\frac{1}{1+R^\pm/\sqrt{\Sigma}}\left\{\frac{1}{2}\ln[\Sigma] + \frac{1}{\zeta}\left(P^+ + P^-\right)\left[1-\Sigma^{-\frac{1}{2}\zeta}\exp(-\psi L)\right] - \alpha_s L\right\}. \quad (4)$$

The maximum achievable output powers are shown Fig. 2c for both configurations. In conclusion, the derived analytical expressions can be used for optimisation of laser system configurations and for saving time in a more challenging modelling that includes analysis of optical phase evolution and effects of dispersion and Kerr nonlinearity. The work was supported by the Russian Science Foundation (project 14-21-00110).

## References


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