Optics Communications 320 (2014) 27–32

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Imperfect narrow filtering in optical links with phase modulation

E.G. Shapiro [a,b], D.A. Shapiro [a,b,*]

[a] Institute of Automation and Electrometry, Siberian Branch, Russian Academy of Sciences, 1 Koptjug Ave, Novosibirsk 630090, Russia
[b] Novosibirsk State University, 2 Pirogov Str, Novosibirsk 630090, Russia

ARTICLE INFO

Article history:
Received 30 October 2013
Received in revised form
26 November 2013
Accepted 5 January 2014
Available online 15 January 2014

Keywords:
Constellation diagram
Gaussian noise
Optical filters
Optical pulses
Phase shift keying
Pattern effect

ABSTRACT

In order to reveal the pattern effect in the optical signal transmission it is studied as a random complex process. The different sources of detection error are studied for quadrature phase-shift keying in the absence of nonlinearity: the error in the rectangular filter width, the finite duration of the initial pulses, the deviation of detection point from the bit interval center. The dispersion and diameter of cloud in the constellation diagram are calculated and shown to be less for longer initial pulses. The error of imperfect optical system is proved to be important at a noise level of 11 dB and more. The result is also applicable for 8-PSK, 16-PSK and higher formats.

1. Introduction

Exploiting the coherence detection and phase-shift keying formats [1] is a very promising way to increase the capacity of fiber communication networks [2]. Nyquist (sinc) pulse shaping provides spectral efficiency close to the theoretical limit [3]. Perfect Nyquist pulses have the rectangular shape in the frequency domain. It allows one to arrange the frequency channels as dense as possible. In time domain the shape of Nyquist pulses is $\mathrm{sinc}(\pi bt)$, where $2\pi b$ is the channel bandwidth. Sinc-like pulses are extended to neighbor bit intervals to the left and to the right. Since they turn to zero in the centers of bit intervals, it is possible to detect each bit separately when we choose the detection point exactly in the center. Nyquist pulses are well-known in electronics, but relatively new in optics.

From a mathematical point of view the Nyquist pulse can be obtained by passing very short pulse (close to δ-function) through the rectangular optical filter of the width $2\pi b$ in concordance with the bit interval $T=1/b$. The application of rectangular filter corresponds to mixing of an optical pulse with Nyquist signal and integration over time, because the product of Fourier transforms is equivalent to the convolution of functions in the time domain. Then it is impossible to get the perfect sinc-like signal, since theoretically it spreads out along the whole time axis, and then all the pulses influence each other. Similarly it is impossible to realize a perfect rectangular filter, since the delay of a pulse passing through the filter must be infinite [4]. The Nyquist signals in a non-ideal optical system are one of the urgent problems of optical communications [5].

In the present paper we consider an optical communication link where the short pulses are passing through the rectangular filter at the transmitter end in order to form the Nyquist shape. The identical rectangular optical filters is applied at the receiver end. In a linear system, in the context of pulse shapes, the signal passing through two rectangular filters is equivalent to passing through one filter with the minimal bandwidth. In the frequency domain the filtering means multiplication by the transfer function, then only the less width enters the result. Then one can consider the line with identical input and output rectangular filters with minimal width. The following parameters influencing the pulse shape are treated: the finite duration of pulses at start, the variation of rectangular filter bandwidth at the receiver end, and the deviation of the detecting point from the center of bit interval. We analyze the contribution of each factor into the coordinates in the constellation diagram. We compare the effects of noise and imperfect optical system. We find the signal-to-noise ratio (SNR) for which the effect of imperfect optical system occurs greater than that of the noise.

The filter decreases the noise of amplifiers and splits the link bandwidth by individual channels. At the same time the narrow filter changes the shape of pulses broadening them in the time

* Corresponding author at: Institute of Automation and Electrometry, Siberian Branch, Russian Academy of Sciences, 1 Koptjug Ave, Novosibirsk 630090, Russia.
E-mail address: shapiro@iae.nsk.su (D.A. Shapiro).

0030-4018/$ - see front matter © 2014 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.optcom.2014.01.008

domain [4,6]. The pulses overlap, and then the nearest (adjacent) pulses and more distant pulses influence the signal. The result depends on the realization of the bit sequence. This effect (the pattern effect) is a serious problem in high-speed all-optical communications [7].

To include all the possible patterns the sequence has to be modeled as a random process. For the amplitude modulation (on/off keying format) the statistical approach is applied in paper [8]. Here we consider the quadrature phase-shift keying (QPSK) format. The coordinate of given pulse on complex plane can also be considered as a random complex number. The aim of present paper is to find the domain of values and dispersion of this random number. In Section 2 we introduce the possible sources of the error, describe the constellation diagram, the Nyquist signal and the optical system. Section 3 presents the results of simulation: root-mean-square and marginal errors on the complex plane.

2. Sources of error

2.1. Duration of initial pulse

In phase-shift keying QPSK format four values are used for coding, placed equidistantly in the unit circle. We chose the values 0, $\pi/2$, π, $3\pi/2$, a bit pair corresponds to each value. We assume pairs "00", "01", "10", "11", respectively. The sequence of optical pulses is defined by formula $\sum_n c_n E_n(t)$, where $E_n(t) = E(t-nT)e^{-i\omega_c t}$ is complex electric field, n is the number of bits, t is the time, T is the duration of a bit interval, ω_c is the carrier optical frequency, $E(t)$ is the profile of an individual pulse. For pulses with profile $E(t) = A\exp(-t^2/2T_0^2)$, where T_0 is the pulse width parameter, A is a coefficient, the duration is determined relation $W = 1.67T_0$. For QPSK format the coefficients c_n possess the values $c_n \in \{1, i, -1, -i\}$, $i = \sqrt{-1}$.

Mathematically the pulse transmission through the rectangular filter is realized by multiplication of its Fourier transform by rectangular function:

$$B(\omega) = \begin{cases} 1, & |\omega| < \pi b, \\ 0, & |\omega| > \pi b, \end{cases}$$

where b is the spectral width of filter. For the Gaussian pulse with zero phase the profile after the rectangular filtering is given by the formula:

$$\tilde{E}(t) = Ae^{-t^2/2T_0^2}\,\mathrm{Re}\left[\mathrm{erf}\left(\frac{\pi T_0 b}{\sqrt{2}} + \frac{it}{T_0\sqrt{2}}\right)\right]. \quad (1)$$

Here $\mathrm{erf}(x) = 2\int_0^x e^{-t^2}\,dt/\sqrt{\pi}$ is the error function [9]. To study the effect of neighbor pulses let us use value $\tilde{E}(\delta_0)$ as a normalization factor, where δ_0 is the deviation of detecting point from the center of bit interval, then the single pulse is located in point (1,0).

Let us calculate the influence of a remote pulse that is separated by l bit intervals from the pulse with $n=0$. The normalized distortion is

$$\varepsilon_l = \frac{e^{-\tau^2/2T_0^2}\,\mathrm{Re}\left[\mathrm{erf}\left(\frac{\pi T_0 b}{\sqrt{2}} + \frac{i\tau}{T_0\sqrt{2}}\right)\right]}{\mathrm{Re}\left[\mathrm{erf}\left(\frac{\pi T_0 b}{\sqrt{2}} + \frac{i\delta_0}{T_0\sqrt{2}}\right)\right]}, \quad (2)$$

where b is the bandwidth of filter, $\tau = lT + \delta_0$. The effect of $2k$ neighbor pulses is given by the sum

$$\xi(k) = 1 + \sum_{l=1}^{k} (\varepsilon_l c_l + \varepsilon_{-l} c_{-l}). \quad (3)$$

where c_l and c_{-l} are statistically independent values. Coefficient c_l posses values $\{1, i, -1, -i\}$ with probability 1/4. If $\delta_0 = 0$, then $\varepsilon_l = \varepsilon_{-l}$.

After filtering the coordinate of a pulse is a complex random number. The coordinates depend on the neighbor pulses. If we take into account k neighbor pulses to the left and k to the right,

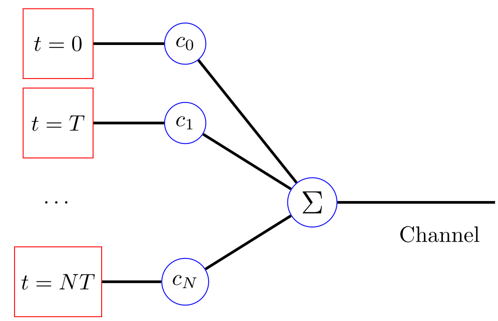

Fig. 1. Schematic diagram of the transmitter end of optical system.

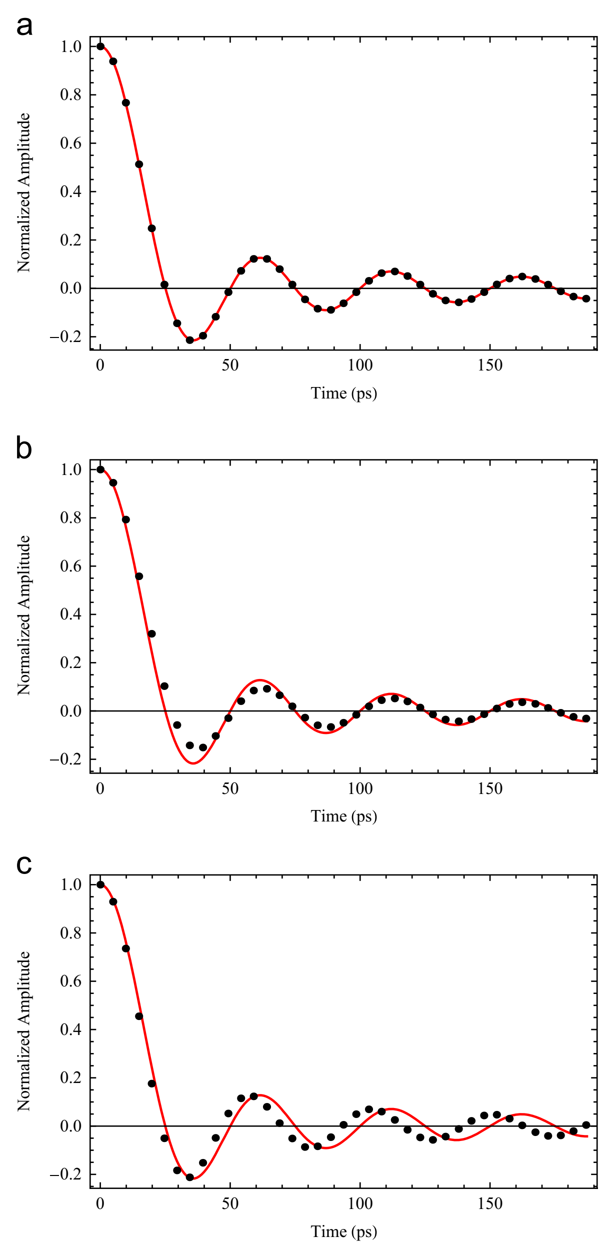

Fig. 2. Comparison of signal obtained from narrow pulse (pulse duration is W=2.5 ps) with the help of rectangular filter (with bandwidth b=40 GHz) (filled circles) with pure Nyquist pulse (solid line) (a). The same for longer pulse (pulse duration W=12.5 ps) (b) and for wider filter (bandwidth b=43 GHz) (c).

then the domain of possible values of the coordinates belongs to the square with center $(1,0)$ and diameter $D=2\sum_{l=1}^{k}(|\varepsilon_l|+|\varepsilon_{-l}|)$. The mathematical expectation of random value $\xi(k)$ is equal to unity: $M\xi(k)=1$.

An important characteristics of a random value is its dispersion. The dispersion of complex random number is given by relation $D\xi=M|\xi-M\xi|^2$. For $\xi(k)$ the dispersion is

$$\sigma^2=D\xi(k)=\frac{1}{4}\sum_{l=1}^{k}(\varepsilon_l^2+\varepsilon_{-l}^2). \quad (4)$$

In a real optical system a finite number of neighbor pulses to the left and to the right should be taken into account. To choose the number of pulses one has to know the specifics of optical filter realization. We calculate the dispersion in Section 3 for $k=4$.

2.2. Nyquist signal

The signal achieves the maximum bit interval utilization [3] when its shape is

$$\mathrm{sinc}(\pi bt)=\frac{\sin\,(\pi bt)}{\pi bt}, \quad b=\frac{1}{T}. \quad (5)$$

The Nyquist profile can be obtained by rectangular filtering of δ-function. At zero phase the perfect sinc-like signal turns to zero at the centers of neighbor bit interval and hits the constellation diagram in point $(1,0)$.

However, the shape of signal can deviate from perfect profile (5) if the initial Gaussian pulse is insufficiently narrow. The error is possible when parameter T_0 is not infinitely small, the width of rectangular optical filter b differs from T^{-1} (greater or less) or the shape of filter deviates from the rectangular one. Furthermore the detection time error is possible when the phase measurement at the receiver end is carried out not exactly in the center of bit interval. The deviation from the ideal optical system transforms the point in the constellation diagram into a cloud on the complex plane.

2.3. Optical system

We consider the following optical system. Transmitter generates Gaussian pulses that pass through the rectangular filter. Then the channels are mixed. At the receiver end the channels are separated. The signal is mixed with the Nyquist signal and integrated over time for determination of the coefficients c_n of the bit sequence (coherence detecting). From a mathematical point of view the convolution with function (5) is equivalent to passing through a rectangular filter of width $b=1/T$.

Scheme of the transmitter end is shown in Fig. 1. Rectangles denote generation of Nyquist signals with fixed time delay

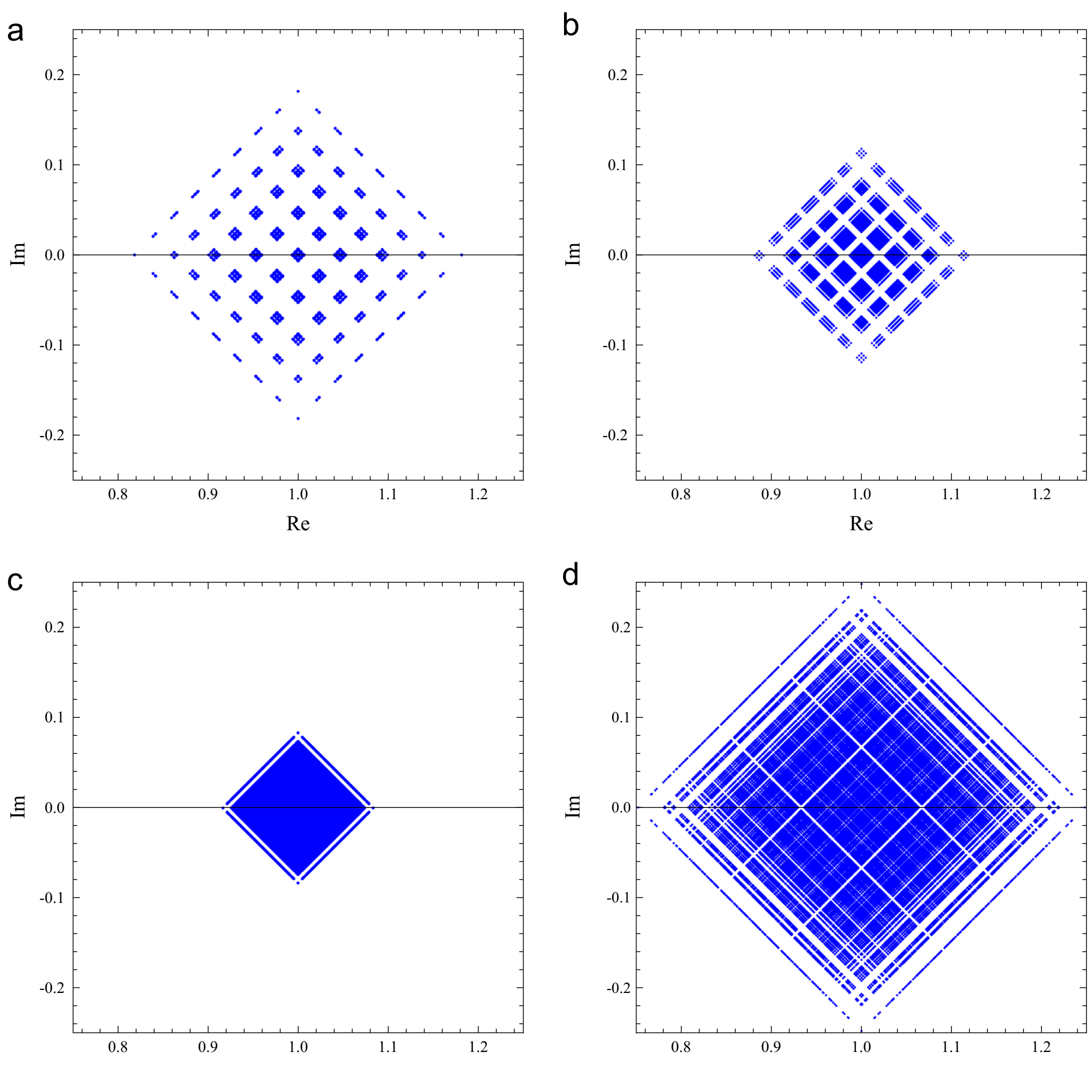

Fig. 3. Distribution of positions of detected pulses in the constellation diagram: $b=41$ GHz, $W=2.5$ ps, $\delta_0=0$ (a), $b=41$ GHz, $W=6.67$ ps, $\delta_0=0$ (b), $b=40$ GHz, $W=2.5$ ps, $\delta_0=0.5$ ps (c), $b=40$ GHz, $W=2.5$ ps, $\delta_0=1.5$ ps (d).

$t=0,T,\ldots,NT$. Small circles correspond to modulation of each signal by complex coefficient $c_n, n=0,1,\ldots,N$. Symbol Σ marks mixing all the signals and transmission to the communication line. The receiver end of the optical system has the same structure. Multiplexer Σ is replaced by the coherent detector (splitter), where the signal is mixed with the shifted Nyquist pulse and integrated over time. Each coefficient at the receiver is determined by formula

$$c_k = \frac{b}{A}\int_{-\infty}^{\infty} \tilde{E}(t)\,\mathrm{sinc}\,[\pi b(t-kT)]e^{i\omega_c t}\,dt. \qquad (6)$$

The infinite limits of integration enable one to use the exact orthogonality relation

$$\int_{-\infty}^{\infty} \mathrm{sinc}\,[\pi b(t-mT)]\,\mathrm{sinc}\,[\pi b(t-nT)]\,dt = \frac{\delta_{mn}}{b},$$

where $\delta_{mn} =$ 1, if $m=n$, or 0 else, is the Kronecker delta-symbol.

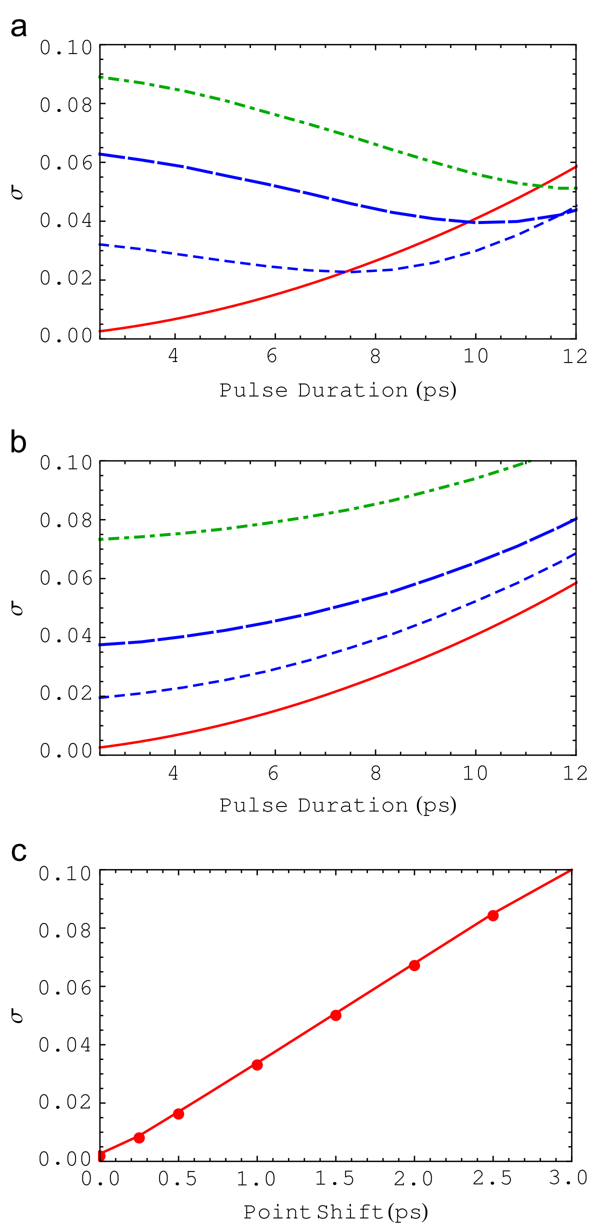

Fig. 4. RMS deviation for wider filter (a): $b=40$ (solid line), 41 (dashed), 42 (long dashes), 43 GHz (dot-dashed). RMS deviation for the narrower filter (b): $b=40$ (solid line), 39.5 (dashed), 39 (long dashes), 38 GHz (dot-dashed). RMS deviation as a function of the detection point shift (c) at $b=40$ GHz, $W=2.5$ ps.

3. Numerical simulation

We vary the width b of optical filter, the pulse parameter T_0 and the deviation δ_0 of the detection point from the bit interval center. The results of simulation follow.

Fig. 2(a, b) illustrates the influence of pulse width parameter T_0. Fig. 2(a) shows that if the pulse duration $W=2.5$ ps is much less than the bit interval $T=25$ ps, then the shape of signal after rectangular filtering is very close to the Nyquist profile. If the pulse is not sufficiently narrow, the shape in Fig. 2(b) essentially departs from the perfect Nyquist profile. Fig. 2(a, c) describes the influence of the filter width. Fig. 2(c) demonstrates that for short pulse the deviation from the Nyquist profile occurs at 7.5% variation in the width. Moreover, as distinct from the Nyquist profile, the zeros of the function are no longer equidistant.

The constellation diagram on the complex plane at the receiver end is shown in Fig. 3. Fig. 3(a) depicts the distribution after the rectangular filter with $b=41$ GHz for a pulse of duration $W=2.5$ ps. Fig. 3(b) refers to $b=41$ GHz, $W=6.67$ ps. The detecting point for both subfigures in the first string is the bit interval center. If the detecting takes place not exactly in the center of bit interval, then the coordinates of pulses on the complex plane are also the random numbers. The second string in Fig. 3 illustrates the effect of detecting point error. The detecting point is shifted from the center by $\delta_0=0.5$ ps (c), or by 1.5 ps (d). It is obvious that the diameter of cloud grows fast with the shift δ_0.

Fig. 4 shows the dependence of root-mean-square (RMS) deviation σ for the pulses on imperfectness factors: inaccurate filter bandwidths and shifts of detection point. Fig. 4(a) shows the RMS deviation for the wider filters, Fig. 4(b) shows it for the narrower ones. For wider filter calculation demonstrates also that the dispersion has a minimum. Fig. 4(c) demonstrates the linear increase in σ with the shift δ_0.

The shape of cloud on complex plane is a square, as illustrated by Fig. 3. The marginal deviation of a point from the center for square is half of the diameter $D/2$ (half of the diagonal). The maximum deviation is important characteristics of the detector. The maximum deviation is shown in Fig. 5. Fig. 5(a) shows the maximal deviation for the wider filters, Fig. 5(b) shows it for the narrower ones. Wider filter calculation also demonstrates that the deviation has a minimum. If the initial pulses are not too short and the filter is a little wider than 40 GHz, then the constellation diagram occurs more compact. For example, at $b=41$ GHz the value $D/2$ remains within 1/10 for almost all the values of pulse duration.

Fig. 5(c) demonstrates that even a small shift of the detection point substantially increases the dimension of cloud. Although the dispersion in Fig. 4(c) is not very large, the diameter of cloud in Fig. 5(c) is much greater. That means the existence of a few “bad” bit sequences resulting with much more deviation from point (1,0) than the average RMS deviation. An example of “bad” sequence is presented in Table 1. In the middle of the sequence there is the unit coefficient $c_0=1$, where $c_l\varepsilon_l<0$, $l=\pm1, \pm2, \pm3, \pm4$. This sequence corresponds to the left vertex of the square on the complex plane. Alternative “bad” sequences can be obtained after the following procedure: let us choose a subsequence and multiply all its elements by i. The factor i can be changed by $-i$ for all the elements in order to obtain the complex conjugated “bad” sequence. Thus there are 2^9-1 “bad” bit sequences. On the complex plane these sequences correspond to the left sides of square. The bisectors of the first and the fourth coordinate quadrants are lines restricting the decision-making domain of “0,0” bit pair. Then the left sides of square are the most risky in the recognition process.

In addition to considered above there is an important factor of signal degradation in the optical communication line, the noise of

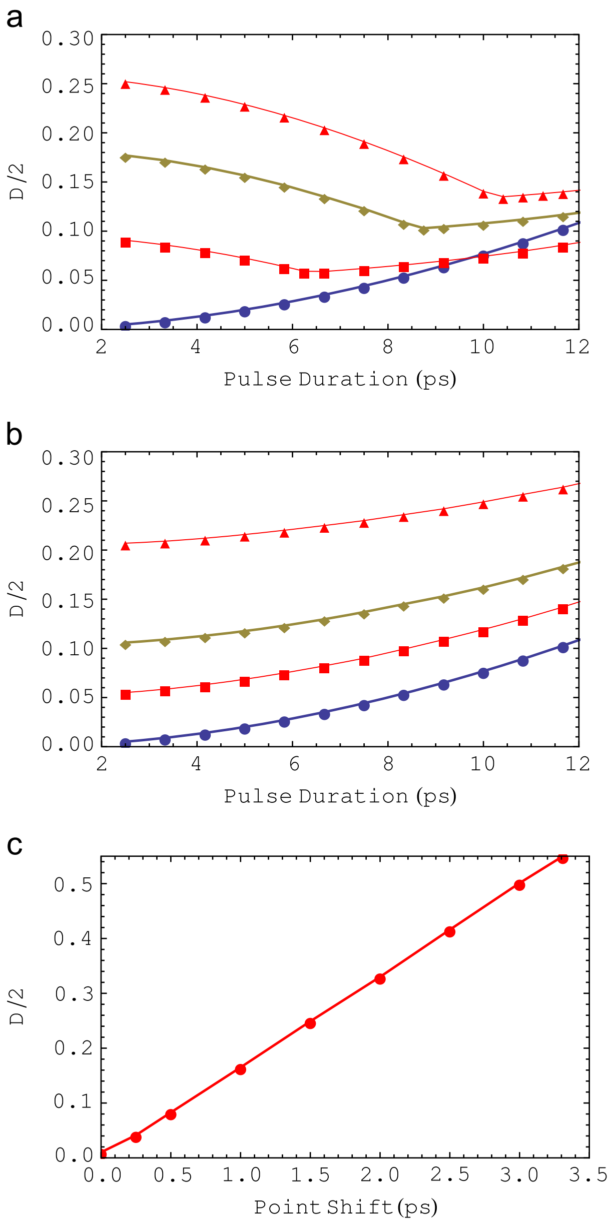

Fig. 5. Half a diameter of the cloud (marginal deviation) in the constellation diagram as a function of pulse duration W. For the wider filter (a): $b = 40$ (circles), 41 (squares), 42 (diamonds), 43 MHz (triangles). For the narrower filter (b): $b = 40$ (circles), 39.5 (squares), 39 (diamonds), 38 GHz (triangles). Diameter of the cloud as a function of the detection point shift at $b = 40$ GHz, $W = 2.5$ ps (c).

Table 1
Coefficients c_n of "bad" sequence.

c_{-4}	c_{-3}	c_{-2}	c_{-1}	c_0	c_1	c_2	c_3	c_4
−1	1	−1	1	1	−1	1	−1	1

amplifiers. In the absence of noise the pure Nyquist signal with zero phase hits the complex plane in point (1,0), when it is normalized by the value in the center. When we add the Gaussian noise the point is converted into a cloud. Fig. 6 displays the average dimension of this cloud as a function of SNR. Comparison with Fig. 4 proves that the imperfect pulse duration, inaccuracy in the filter width and deviation of the detecting point gives the same effect as the Gaussian noise in the perfect optical system at SNR below 16–18 dB.

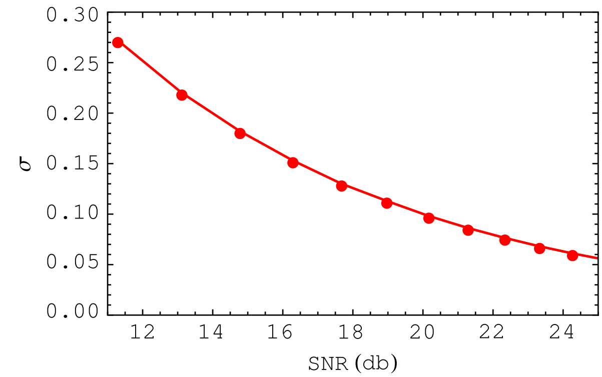

Fig. 6. RMS deviation as a function of signal-to-noise ratio.

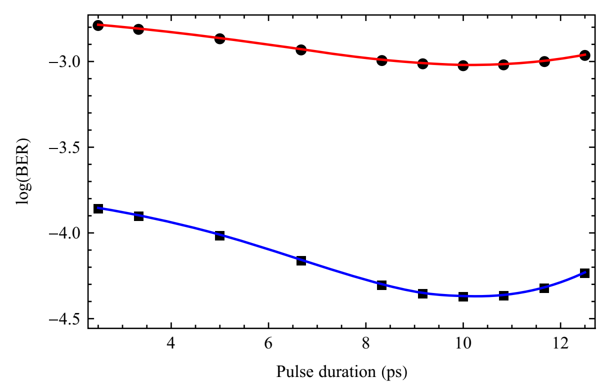

Fig. 7. BER as a function of Pulse duration W for SNR = 10.65 (circles) and 12.6 dB (squares).

In an imperfect optical system with Gaussian noise the coordinates of pulses in the constellation diagram depend both on the neighbor pulses and on the level of noise. The more RMS deviation from point (1,0), the greater probability of error detection is. Fig. 7 shows the dependence of BER (bit-error rate) on the initial pulse duration for channels with the noise level SNR = 10.65 and 12.6 dB. The optical filter width is $b = 42$ GHz, the detection occurs in the bit interval center. It is obvious that at SNR ≈ 11 dB the signal quality can be improved utilizing wider pulses. This conclusion is in agreement with Figs. 4(a) and 5(a). As for the quantitative values, the system without filtering, i.e. perfect optical system with the noise SNR = 10.65 and 12.6 dB, the bit-error rate is $\log_{10}(\mathrm{BER}) = -3.2$ and -4.8, respectively.

Note that the result can be extended to the higher formats. The number of different values of phase is equal to the number of vertices of the regular polygon on complex plane. The distribution in constellation diagram has the shape of regular polygon, too. For 8-PSK it is a regular octagon etc. The diameter of an octagon and the root mean square deviation are given by the same formulas as the square for QPSK format.

4. Conclusions

The location of detected pulse in the constellation diagram is considered as a random complex number. We clear the domain of its values on the complex plane and its broadening due to the pattern effect. For QPSK format it is a square. The dispersion of random normalized value $\xi(k)$ and the diameter of cloud on the complex plane are calculated analytically. The longer pulses are shown to be preferable for a line with imperfect filtration (floating bandwidth of the rectangular filter). For the longer pulses both the dispersion and diameter of cloud are less, since the errors partially cancel each other. The most “risky” sequences are revealed corresponding to the left sides of the square in the constellation diagram. The new effects of imperfect phase detection are compared with known effect of Gaussian noise and shown to be important for the noise level nearly 11 dB and more.

Acknowledgments

The authors would like to thank Prof. S.K. Turitsyn for helpful discussions. This work is supported by Grant of Ministry of Education and Science of the Russian Federation (agreement No. 14.B25.31.0003), the Government program of the leading research schools NSh-2979.2012.2, and by the Russian Academy of Sciences: program # 24 of its Presidium, grants from its Division of Physical Sciences and Siberian Branch.

References

[1] A. Gnauck, P. Winzer, J. Lightwave Technol. 23 (1) (2005) 115.
[2] R. Essiambre, G. Kramer, P. Winzer, G. Foschini, B. Goebel, J. Lightwave Technol. 28 (4) (2010) 662.
[3] R. Schmogrow, M. Winter, M. Meyer, D. Hillerkuss, S. Wolf, B. Baeuerle, A. Ludwig, B. Nebendahl, S. Ben-Ezra, J. Meyer, M. Dreschmann, M. Huebner, J. Becker, C. Koos, W. Freude, J. Leuthold, Opt. Express 20 (2012) 317.
[4] E.G. Shapiro, D.A. Shapiro, Electron. Lett. 49 (4) (2013) 279.
[5] C. Liu, J. Pan, T. Detwiler, A. Stark, Y.-T. Hsueh, G.-K. Chang, S.E. Ralph, Opt. Express 21 (7) (2013) 8342.
[6] O.V. Belai, D.A. Shapiro, E.G. Shapiro, Quantum Electron. 36 (9) (2006) 879.
[7] E. Shapiro, M. Fedoruk, S. Turitsyn, Electron. Lett. 37 (19) (2001) 1179.
[8] E.G. Shapiro, M.P. Fedoruk, S.K. Turitsyn, Electron. Lett. 40 (22) (2004) 1436.
[9] F. Olver (Ed.), NIST Handbook of Mathematical Functions, NIST and Cambridge University Press, New York, 2010.